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Computer-aided optimization of grid design for high-power lead–acid batteries

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Abstract

Several high-power lead–acid batteries have been developed for automotive applications. A computer-aided optimization (CAO) technique has been used to obtain a low-resistance grid design. Unlike conventional computer simulation, the CAO technique does not require an unduly large number of designs to yield a good result. After introducing a pair of differential equations that are expected to be valid for the optimized design, the grid thickness is optimized by solving the boundary value problem of coupled differential equations. When applied for the grids of JIS B-size batteries, this technique reduces the potential drop of electrical resistance in a electrode by 11–14%.

Keywords: Computer-aided optimization; Grid design; High-power lead–acid battery; Electrical resistance

1. Introduction

The power required from automotive lead–acid batteries has been increasing over the past 10 decades. Moreover, battery manufacturers nowadays are required to produce smaller automotive batteries that contain less lead and can fit into compact engine compartments. To respond to these demands, our group has developed a series of high-power automotive batteries (e.g., the 110D26 [1]) that can replace larger predecessors. This has been achieved through the use of several techniques that included a computer-aided engineering (CAE) technique to modify the grid designs of the positive electrodes [2–6]. The advantage of the CAE technique is its ability to obtain the potential distribution in an electrode for a given grid design. Yet, due to the high complexity of grid designs, the CAE technique usually requires an unduly large number of designs to yield a good result.

More recently, the newly developed computer-aided optimization (CAO) technique has been successfully applied to determine optimized designs automatically in several design fields [7,8]. Given the considerable time required to determine grid designs for large numbers of modified grids, the CAO technique seems suitable for application. The target of this study was to develop a method to design low-resistance grids. The grid-design process consists of the arrangement and determination of the thickness of each member. In this study, the thickness of each member was optimized and the arrangement was determined without the use of a computer.

2. Theory

2.1. Forming the problem

A solution was found to the boundary value problem of coupled differential equations that were expected to be valid for the optimized design.

Consider a positive electrode of a lead–acid battery made of a grid and active material (see Fig. 1). The width and height of the electrode are denoted as $X_{\max}$ and $Y_{\max}$, respectively. The resistivities of the grid and active material are shown in Table 1. The current density into the solution is assumed to be uniform at all points except at the lug, where the current

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Nomenclature

A	area of electrode without lug (cm^2)
$d(x, y)$	thickness of grid (cm)
E	electrode potential (V)
$E^{0\prime}$	formal potential (V)
F	Faraday constant ($C\,mol^{-1}$)
h	height of the electrode without lug (cm)
I	current per electrode (A)
j	current density ($A\,cm^{-2}$)
j_0	exchange current density ($A\,cm^{-2}$)
n	the number of electrons transferred
R	universal gas constant ($J\,mol^{-1}\,K^{-1}$)
T	temperature (K)
X_{max}	width of the electrode (cm)
Y_{max}	height of the electrode (cm)

Greek letters

α	transference factor
Δ	distance between two points side by side (cm)
$\rho(x, y)$	resistivity of electrode material
$\phi(x, y)$	potential drop (V)

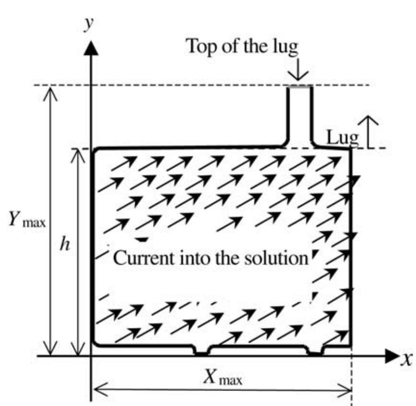

Fig. 1. Positive electrode of lead–acid battery.

density is neglected. The resistivity of the electrode is considered to be a function of the location within the electrode. The charge conservation equation can be expressed as follows:

$$\left(\frac{d}{dx}+\frac{d}{dy}\right)\left[\frac{d(x, y)}{\rho(x, y)}\left(\frac{d\phi(x, y)}{dx}+\frac{d\phi(x, y)}{dy}\right)\right]=-j \quad (1)$$

with

$$j = IA^{-1}(0 \le y \le h), 0(h < y \le Y_{max})$$

Table 1
Resistivities (Ω cm) of materials used in calculation

Grid (lead)	2.06×10^{-5}
Positive active material	1.05×10^{-2}

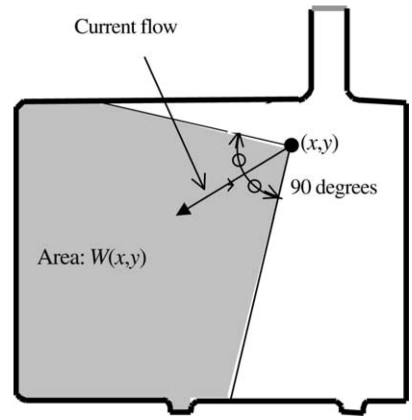

Fig. 2. Definition of $W(x, y)$ in this study.

where $d(x, y)$, $\rho(x, y)$, $\phi(x, y)$, and j are the thickness of grid, the resistivity of the electrode material, the potential drop in the electrode, and the normal current density of the electrode, respectively; h the height of the electrode without the lug; I the current per electrode; A the area of the electrode without the lug. The current I in this study is taken as 100 A. This is assumed to be a proper value since the current at an engine start can be over 500 A and some batteries contain five positive electrodes per cell.

In addition to the above treatment, consideration is also given to a condition that will be valid for the optimized grid thickness. An evaluation was made of the grid design with the calculated potential drop of the electrode, that is, the total heat of the joule effect divided by the square of the current (100 A). Two parameters were used to estimate the effect of a thickness change, at any given point, on the potential drop of the electrode namely: (i) the change in the potential drop at the chosen point, expressed as $d(x,y)^{-1}(d/dx + d/dy)\phi(x,y)$, and (ii) the area of the section where the thickness change can affect the potential drop. Given that the design can be optimized by transferring the lead from an ineffective location to an effective location, the thickness change will have a uniform affect on the potential drop in the optimized design, as follows:

$$d(x, y)^{-1}\left(\frac{d}{dx}+\frac{d}{dy}\right)\phi(x, y) \times W(x, y) = \text{constant} \quad (2)$$

where $W(x, y)$ is the area surrounded by the outline of the electrode and the two lines (see Fig. 2) that cross at (x, y) and at $\pm 45^\circ$ to the current flow at (x, y).

The boundary conditions are expressed as follows:

$$x = a(0 \le \alpha \le X_{max}),$$
$$y = y_{max}(x = a) = Y_{max}: \quad \phi = 0 \quad (3)$$

$$x = a(0 \le \alpha \le X_{max}),$$
$$y = y_{max}(x = a) \ne Y_{max}: \quad \frac{d\phi}{dy} = 0 \quad (4)$$

$$x = a(0 \le \alpha \le X_{\max}), \quad y = y_{\min}(x = a): \quad \frac{d\phi}{dy} = 0 \quad (5)$$

$$y = a(0 \le \alpha \le X_{\max}), \quad x = x_{\max}(y = a): \quad \frac{d\phi}{dx} = 0 \quad (6)$$

$$y = a(0 \le \alpha \le X_{\max}), \quad x = x_{\min}(y = a): \quad \frac{d\phi}{dx} = 0 \quad (7)$$

where $y_{\max}(x = a)$ and $y_{\min}(x = a)$ are the maximum and minimum values of y on the electrode in line $x = a$, and $x_{\max}(y = a)$ and $x_{\min}(y = a)$ are the maximum and minimum values of x on the electrode in line $y = a$.

2.2. Calculating the problem

The boundary value problem of coupled differential equations can be solved for the optimization of the grid thickness by an iterative method. The following approximations are used for the calculation of Eq. (1)

$$\sum_m \phi(x_{n,m}, y_{n,m}) R_{n,m}^{-1} - \phi(x_n, y_n) \sum_m R_{n,m}^{-1} = -j\Delta^2 \quad (8)$$

with

$$R_{n,m} = \frac{1}{2}(\rho(x_n, y_n) d(x_n, y_n)^{-1} + \rho(x_{n,m}, y_{n,m}) d(x_{n,m}, y_{n,m})^{-1}) \quad (9)$$

where n $(1 \le n \le N_{\max})$ is the number of points on the electrode; (x_n, y_n) and $(x_{n,m}, y_{n,m})$ $(m = 1–4)$ are the nth point on the electrode and the points next to the nth point; Δ is the distance between two points side-by-side. The total number of points on the electrode is about 2×10^5. When the nth point is on the outline of the electrode, the number of points next to the nth point becomes less than 4. Eq. (8) is solved using the Gauss–Seidel method to obtain the potential distribution in the electrode.

Next, the thickness of the grid was varied in order to satisfy Eq. (2), under the assumption that the amount of lead is preserved. In this study, optimization's made of the inner grid members, the top frame, and the vertical frame far from the tab. Maximum and minimum values were set for the thickness for the handling, corrosion, and dimension of the cell—three factors that constrain the design of the thickness. The variation of the thickness is calculated using the following equation

$$d(k + 1)(xn, yn) - d(k)(xn, yn) \quad (10)$$

The calculation of the potential distribution and modification of the thickness are iterated until the thickness distribution converges at 0.01%.

The initial grid design is written in an XPM format bitmap file and a text file that describes the correspondence of colours with the thickness and the classification of the divisions mentioned below. The division of each colour in the bitmap file is classified into three divisions: one where no alteration in thickness is permitted, one where an increase in thickness is permitted, and one where an increase and a decrease in thickness are permitted. The lug, left frame (away from the lug) and bottom frame are allotted to the first division; the top frame and right frame (close to lug) are allotted to the second division; and the inner grid members are allotted to the third division. The thickness is not optimized for each point, but for each coloured area corresponding to a thickness. The program was written in C++ language and executed on a Windows NT Version 4 workstation (CPU: DEC Alpha 21164 533 MHz). Computation of a single optimization problem required less than 4 h.

3. Results and discussion

3.1. Effect of optimization on potential drop

An attempt was made to modify the conventional grid shown in Fig. 3 (grid A) for use in JIS B-size batteries (1.45 mm thick, 105 mm wide, 111 mm high of h in Fig. 1, and 36 g in weight). The parameters used for the optimization are listed in Table 2. The potential in the electrode before the optimization falls near the lug, as shown in Fig. 4 (A). The potential distribution after the optimization is shown in

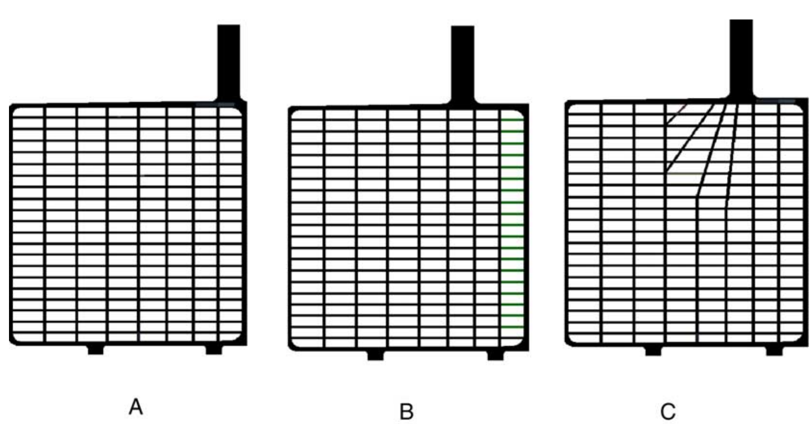

Fig. 3. Grids for JIS B-size batteries. (A) Conventional grid; (B) inner lug grid; (C) radial grid with inner lug.

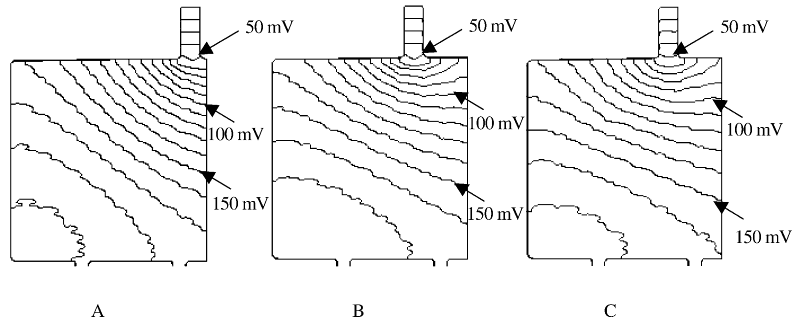

Fig. 4. Potential distribution before optimization. (A) Conventional grid; (B) inner lug grid; (C) radial grid with inner lug. Current: 100 A per electrode.

Table 2
Parameters used for grid optimization in B-size battery

Maximum thickness (cm)	3.0×10^{-1}
Minimum thickness (cm)	4.15×10^{-2}

Table 3
Calculated potential drop for 100 A per electrode (mV)

Grid	Before optimization	After optimization	Effect of optimization
A	161	143	18 (11%)
B	147	127	20 (14%)
C	138	120	18 (13%)
Grid in 36 V battery	102	94	8 (8%)

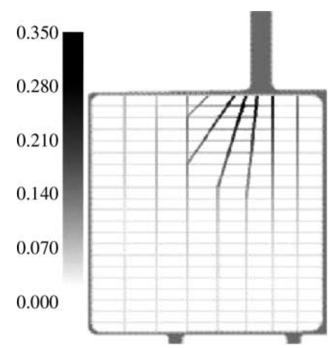

Fig. 6. Optimized thickness of grid C.

Fig. 5(A). The intervals of the isopotential lines widen somewhat compared with the interval in Fig. 4(A). The calculated potential drop of the electrode (Table 3) decreases from 161 to 143 mV as a result of this technique.

Though the current density is treated as uniform in this study, a high rate of discharge will limit the area in which the electrode reaction occurs when a stricter treatment is applied. In this case, the relationship between the current density j and the potential drop ϕ is expressed as follows:

$$j = j_0 \exp\left(\left(\frac{\alpha n F}{RT}\right)(E - E^{0'}) - \frac{\alpha n F \phi}{RT}\right) \quad (11)$$

where j_0 is the exchange current density; α the transference factor; n the number of electrons transferred; E the electrode potential; $E^{0\prime}$ the formal potential of the electrode reaction. When the current density is large at a point on the electrode, no electrode reaction can occur in areas where the potential drop exceeds that on the electrode by more than $RT/\alpha nF$. Thus, the areas with large current densities will move away from the lug as time passes. Since $RT/\alpha nF$ equals 26 mV when $T = 298.15$ K, $\alpha = 0.5$ and $n = 2$, the improvement of grid A, 18 mV, viz., is sufficient.

The optimization can be expected to increase the area where the electrode reaction occurs at high rate of discharge. The current density will thus be small, and the overpotential of the electrochemical reaction will also become small. Though only the active material near the surface is used at high rates of discharge, the small current density may increase the available active material.

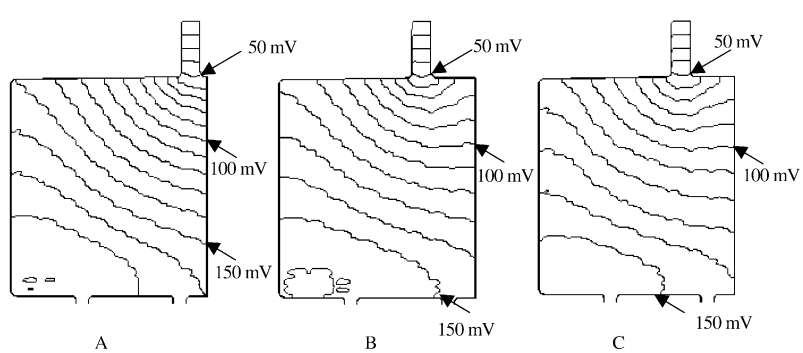

Fig. 5. Potential distribution after optimization. (A) Conventional grid; (B) inner lug grid; (C) radial grid with inner lug. Current: 100 A per electrode.

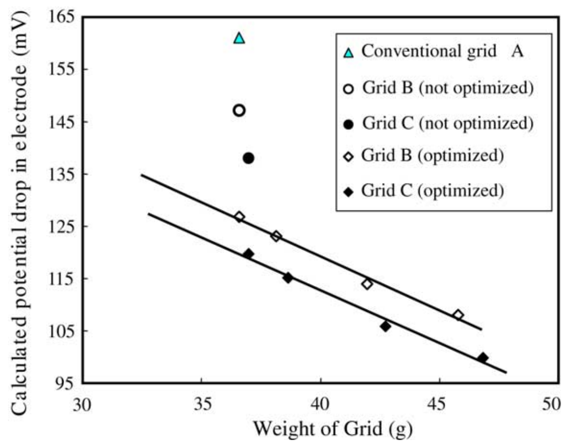

Fig. 7. Calculate potential drop in electrodes with various grid weights. Current: 100 A per electrode.

3.2. Comparison with some modifications of grid configuration

An inner lug grid (Fig. 3, grid B) and radial grid with an inner lug (Fig. 3, grid C) are generally effective at reducing the electric resistance. The potential drop of B-size electrodes before and after the optimization, are shown in Figs. 4 and 5, respectively. The values of the calculated potential drop of the electrode are listed in Table 3. In the grid configurations with an inner lug or a radial grid, the potential drop is reduced by 14 or 9 mV, respectively. These reductions are smaller than the 18 mV reduction obtained by the optimization. Combining these preferable grid configurations and the thickness optimization, we can reduce 41 mV (i.e., 25%) of the potential drop at grid A. As shown in Fig. 6, the optimized grid has very thick members near the lug. Thick members near the lug and thin members far from the lug lead to small potential drops with the use of only small amounts of lead. The potential drops in electrodes calculated with grids of various weights are given in Fig. 7. From the slope of the plot it is found that this optimization technique can reduce the grid weight by 0.5 g when the potential drop increases by 1 mV, and reduces the potential drop by 2 mV when the grid weight increase by 1 g. This technique can also be applied to reduce the amount of lead.

3.3. Application to 36 V battery

The above technique was applied to the grid shown in Fig. 8 (2.2 mm thick, 72 mm wide, 150 mm high, and 79 g in weight) which was used in a prototype of a 36 V battery [9] for 42 V automotive systems. Since the current at an engine start for this system can be about 250 A and the 36 V battery contain four positive electrodes per cell, the current per electrode (63 A), is somewhat smaller than that in 12 V automotive batteries. In designing the grid of the 36 V battery, importance was attached to both the longevity and the electric conductivity, and the optimization parameter for the

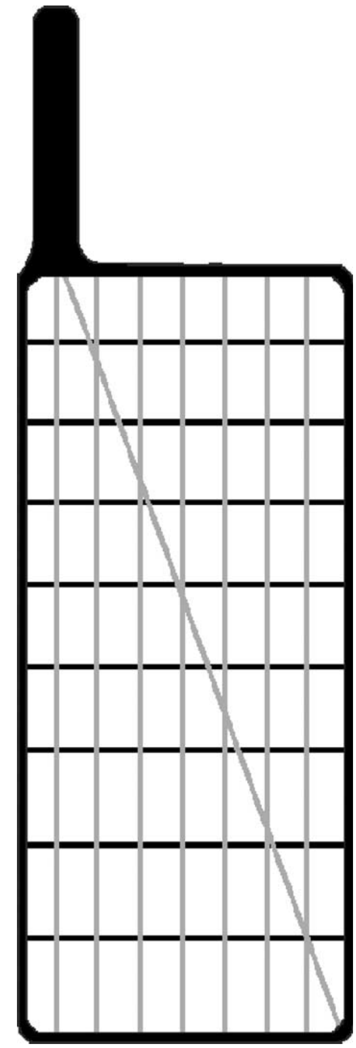

Fig. 8. Grid for 36 V battery.

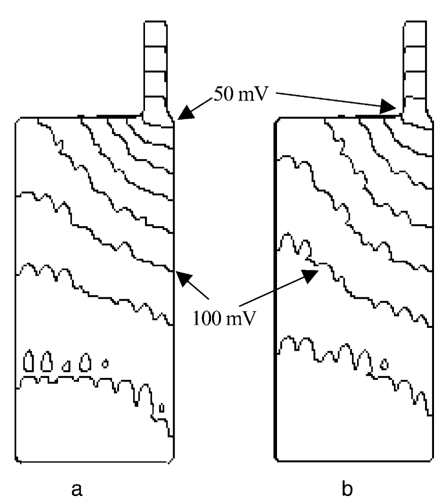

Fig. 9. Potential distribution in 36 V battery (a) before and (b) after optimization. Current: 100 A per electrode.

Table 4
Parameters used for grid optimization in 36 V battery

Maximum thickness (cm)	3.5×10^{-1}
Minimum thickness (cm)	1.8×10^{-1}

minimum value of thickness was set larger than that for the B-size batteries shown in Table 4. The optimization causes an improvement in the potential distribution (Fig. 9), but the calculated potential drop in Table 3 is smaller than that of B-size batteries owing to the large optimization parameter of minimum thickness. Taking account of the small current and the requirement of battery longevity, the effect of optimization may be limited for high-voltage automotive systems.

4. Conclusions

A boundary value problem of coupled differential equations has been solved to design low-resistance grids. While Eq. (2) may not be ideal for the optimization, it has proven to be far more serviceable than the conventional simulation technique in the design of grids. Automotive batteries and other batteries for high-rate use (e.g., uninterruptible power supplies) will be good applications.

The use of this technique allows an increase in the thickness of the important grid members. It can be also used to improve battery longevity.

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